Modern Computational Accelerator Physics

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USPAS January 2015

Space charge computation

Collective effects

- Space charge
  - Describes the interaction of the particles with the electromagnetic field generated by the others particles in the beam.
- Wake field
  - Describes the interaction of the particles with the electromagnetic field generated by the induced current in the vacuum chamber walls.

The terminology can be confusing and sometimes by the term wake or by the term space charge both effects are understood. Even more confusing, in the computing community a space charge solver with closed boundary conditions assumes that the chamber is grounded, thus considering the effect of the image charge and current as induced in a perfect conductor.

Computing space charge effects

- A particle motion through an accelerator is described by the Hamiltonian

$$H = H_0 + H_1$$

where H_0 is the single particle term (the free particle and the interaction with the magnets fields) and H_1 is the interaction with the field generated by the other particles in the beam.

- The map of a step of length s is

$$\mathcal{M}(s) = \mathcal{M}_0(\frac{s}{2})\mathcal{M}_1(s)\mathcal{M}_0(\frac{s}{2}) + \mathcal{O}(s^3)$$

where $\mathcal{M}_0$ is the single particle map and $\mathcal{M}_1$ is the map of the interacting term.
(Higher order Yoshida split-operator approximations can be employed.)

- Computing space charge effects reduces to the calculation of $\mathcal{M}_1(s)$.

Computing space charge effects (continued)

- To compute $\mathcal{M}_1(s)$ the electromagnetic field created by the beam should be determined.
- The main approximation assumed for the calculation of the electromagnetic field is that *the beam is rigid in the reference particle frame.*
- In the beam frame the fields are electrostatic and the potential satisfies the Poisson equation

$$\nabla^2\Psi(x, y, z) = -\frac{\rho}{\epsilon_0}.$$

- Once the field is calculated in the beam frame, it can be Lorentz transformed in the accelerator frame $\mathcal{M}_1$ calculated.
- Alternatively, $\mathcal{M}_1$ can be calculated and applied in the beam frame. Afterwards the beam coordinates are Lorentz transformed back to the accelerator frame.

Computing space charge effects (continued)

- In the beam frame

$$H_1 = q\Psi(x, y, z)$$

and the equation of motions are

$$\frac{dx}{dt} = 0 \implies x_{final} = x_{initial}$$

$$\frac{dp_x}{dt} = -q\frac{\partial \Psi}{\partial x} = qE_x \implies p_{xfinal} = p_{xinitial} + qE_x t$$

Analogous equations can be written for the other directions.

Space charge kicks in the beam frame

- In the beam frame the map $\mathcal{M}_1(\tau)$ implies

$$
\begin{aligned}
x_{final} &= x_{initial} \\
p_{x\ final} &= p_{x\ initial} + qE_x\tau \\
y_{final} &= y_{initial} \\
p_{y\ final} &= p_{y\ initial} + qE_y\tau \\
z_{final} &= z_{initial} \\
p_{z\ final} &= p_{z\ initial} + qE_z\tau
\end{aligned}
$$

- Here τ is the corresponding step length in the beam frame, $\tau = \frac{s\gamma}{\beta c}$

Space charge solvers in Synergia

- They all solve the Poisson equation in the beam frame.
  - Variety of boundary conditions and levels of approximation
- 3D open transverse boundary conditions
  - Hockney algorithm
  - open or periodic longitudinally
- 3D conducting rectangular transverse boundary
  - periodic longitudinally
- 3D conducting circular transverse boundary
  - periodic longitudinally
- 2.5D open boundary conditions
  - 2D calculation, scaled by density in longitudinal slices
- 2D semi-analytic
  - uses Bassetti-Erskine formula
  - σ_x and σ_y calculated on-the-fly

Space charge solvers

- Next we will write a 2D solver with closed conducting rectangular boundary.