Modern Computational Accelerator Physics

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More Background for Accelerator Simulation Techniques plus CHEF

The Reference Particle

- Even using z as the independent variable, tracking particles around an accelerator's design trajectory using global coordinates is excessively cumbersome.
- Tracking becomes dramatically simpler when expressed in terms of differences from the trajectory of a hypothetical *reference particle*. We express all particle coordinates in this way, including the reference particle itself, which tends to have zeros for coordinates.

Transfer Maps

- We can consider the action of the Hamiltonian on the particles as (generally nonlinear) mapping $\mathcal{M}$ which takes the six-dimensional phase space coordinates from P^{in} to P^{out}:

$$P^{out} = \mathcal{M} P^{in}$$

- A very special case is the linear map M which essentially reduces the problem to linear optics.
    - One easily accessible result is that stability of a lattice requires

$$\det M = 1.$$

- Another useful case is an n^{th} order polynomial in the coordinates. We call that a *polynomial* (or sometimes "*higher-order*") *map*.

Symplectic Matrices

(Again, this is lifted from Ryne.)
Let M denote a $2m \times 2m$ matrix. Let J denote the $2m \times 2m$ matrix given by.

$$J = \begin{pmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & J_1 \end{pmatrix},$$

where J_1 is the 2×2 matrix given by

$$J_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The matrix M is said to be a *symplectic matrix* if it satisfies

$$\tilde{M} J M = J.$$

Symplectic Matrices (2)

Symplectic matrices have several important properties that we state here:

1. $\det M = 1$.
2. The eigenvalues of M are real or they occur in complex conjugate pairs.
3. If λ is an eigenvalue of M, then so is $1/\lambda$.
4. The real dimensionality of M (i.e. the number of real parameters necessary to specify an arbitrary $2m \times 2m$ symplectic matrix) is $m(2m + 1)$.
5. The set of $2m \times 2m$ symplectic matrices forms a group, Sp(2m).

CHEF

As we have seen, Synergia uses the CHEF libraries to do independent particle tracking.

- Internally, CHEF tracks particles according to element propagators. These propagators are programmable.
- The default propagator for most elements consists of a series of empty spaces and thin element kicks, constructed to be symplectic.
- CHEF can do internal symbolic manipulations to extract what amounts to a multi-dimensional Taylor series expansion of a

mapping.

- Linear mappings extracted that way are always symplectic.
- Higher-order mappings are generally *not* symplectic, but it is possible to symplectify them.

CHEF Units

Every simulation package uses some sort of scaled system of units.
Synergia uses CHEF's system.
(Below, p_{ref}^{total} is the total momentum of the reference particle, in GeV/c)

- In the fixed-z representation:

x : [meters]
y : [meters]
cdt : $c\Delta t$ [meters]

xp : $\frac{p_x}{p_{ref}^{total}}$ [unitless]
yp : $\frac{p_y}{p_{ref}^{total}}$ [unitless]
dpop : $\frac{\Delta p^{total}}{p_{ref}^{total}}$ [unitless]

- In the fixed-t representation:

x : [meters]
y : [meters]
z : [meters]

xp : $\frac{p_x}{p_{ref}^{total}}$ [unitless]
yp : $\frac{p_y}{p_{ref}^{total}}$ [unitless]
zp : $\frac{p_z}{p_{ref}^{total}}$ [unitless]

Higher-order Symplectic Methods

The Method of Yoshida

H. Yoshida, Phys. Lett. A 150, p. 262 (1990).

Suppose we have a Hamiltonian that can be split into two parts,

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2,$$

and that the differential operators that drive the time evolution of $\mathcal{H}_1$ and $\mathcal{H}_2$ are A and B, respectively. Definining $z = (q, p)$, we have

$$z(\tau) = \exp\left[\tau(A + B)\right] z(0).$$

We seek a set of c_i and d_i such that

$$\exp\left[\tau(A + B)\right] = \prod_{i=1}^{k} \exp(c_i \tau A) \exp(d_i \tau B) + \mathcal{O}(\tau^{n+1})$$

. It is easy to find solutions at first order,

$$c_1 = d_1 = 1,$$

and second order,

$$c_1 = c_2 = \frac{1}{2}, d_1 = 1, d_2 = 0.$$

Yoshida, continued

At fourth order, his method produces

$$c_1 = c_4 = \frac{1}{2(2 - 2^{1/3})}$$

$$c_2 = c_3 = \frac{1 - 2^{1/3}}{2(2 - 2^{1/3})}$$

$$d_1 = d_3 = \frac{1}{2 - 2^{1/3}}$$

$$d_2 = -\frac{2^{1/3}}{2 - 2^{1/3}}$$

$$d4 = 0$$

The sign of d_2 seems weird if you ascribe some physical meaning to the intermediate stages of a time step. Don't do that.

Yoshida, yada yada

Explicitly,

$$\exp\left[\tau(A+B)\right] = \exp(c_1\tau A)\exp(d_1\tau B)\exp(c_2\tau A)\exp(d_2\tau B)\times$$
$$\exp(c_3\tau A)\exp(d_3\tau B)\exp(c_4\tau A) + \mathcal{O}(\tau^5).$$

Assignment 4

- Calculate the horizontal and vertical tunes in the foborodobo32 lattice.
  - You can plot a track of a single turn and count the peaks to get a rough estimate.
  - The tunesim.py script will help write a track at the appropriate places.
  - The tunecalc.py will show you how to read the track data and get the FFT.
- Determine the accuracy with which you can measure the tune.

Assignment 5

- Write your own version of independent-particle tracking for lattices involving only drifts and quadrupoles.
  - There is a version that works for thin quadrupoles in thin_fodo.py and minisyn.py.
- Add a 2nd order symplectic integrator for thick quadrupoles.
  - See how many steps it takes to get accurate results.
- Add a 4th order symplectic integrator for thick quadrupoles.
  - See how many steps it takes to get accurate results.