Electron lens routine in Sixtrack

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electron lens routine

The hollow electron beam is a device which generates a hollow beam of electrons traveling in the same direction of the proton beam. The purpose of this report is to describe the electron lens model which has been implemented in the tracking software SixTrack []. The description of the details about the hollow beam generation and technical specifications will not be detailed here, but can be found in [][][].

1 Electron lens kick

The electron lens is implemented in the code in the thin6d routine. The device is treaded as a thin lens giving a single kick at the middle of its lenght. In this section the value of the kick is derived and the approximation details are described.

In Sixtrack an ideal electron lens is considered, i.e. a cylindrical distribution of electrons with internal radius R_1 and external radius R_2. Different radial profiles are available (uniform of measured profile). For the e.m. field calculation, the finite length cylinder is approximated with an infinite one, which is legitimate since the transversal space of interest is of the order of few millimeters, while the total device lenght is typically $L = 2$ m. Fringe fields are neglected.

Given a total electron beam current I_T and a positive, continuous, normalized function $i(r)$ describing the current profile:

$$\int_0^\infty i(\rho)\,\mathrm{d}\rho = \int_{R1}^{R2} i(\rho)\,\mathrm{d}\rho = 1 \tag{1}$$

we define I_r as the current encompassed by the radius r:

$$I_r = I_T \cdot \int_0^r i(\rho)\,\mathrm{d}\rho = I_T \cdot f(r) \tag{2}$$

In case of azymuthal symmetry the resulting force can be easily calculated by using Gauss and Biot-Savart laws. It follows that the force acting on a proton having a distance r from the electron lens center is:

$$\overline{F}(r) = \frac{I_T\ f(r)\ q_p\ (1 \pm \beta_p \beta_e)}{2\pi\varepsilon_0\ r v_e} \overline{\jmath}_r \tag{3}$$

where q_p is the proton charge, β_p and β_e are the relativistic β for the proton and the electron beam ,and $\overline{\jmath}_r = (\overline{x} + \overline{y})/r$ is the radial inward direction. The plus sign is used when the electron and proton velocities have the same versus. Since in a circular accelerator the maximum divergence is of the order of few 10^4, it follows that:

1. the transverse position of the particle along the electron lens device can be considered constant in first approximation;
2. the length of the trajectory of a proton passing through the electron lens can be approximated with the total electron lens L

Keeping in mind the definition of angular velocity $\omega = \theta/t = (v_p/r)$, the crossing time $t = L/v_p$, and the centrifugal force $(1/r) = F/(mv_p^2)$, it is possible to calculate the integrated kick for a particle which crosses the electron lens at transverse amplitude r:

$$\theta(r) = \frac{2L\ f(r)\ I_T\ (1 \pm \beta_e \beta_p)}{4\pi\varepsilon_0\ r\ (B\rho)_p \beta_e \beta_p\ c^2} \tag{4}$$

where $(B\rho)_p = m_p v_p/q = 3.3356(mv)_p[GeV/c]$ is the magnetic rigidity of the proton beam.

Since the electrostatic force is always directed inward, and the electron beam versus is usually chosen such as the Lorentz force adds up to the electric one, the kick given by the electron lens is always directed inwards (focusing), both for the vertical and for the horizontal plane. If we consider what would be the effect of a single kick of the electron lens on the unperturbed harmonic motion of a particle in a linear machine, it is clear that a single inward kick can both increase (quadrant II and IV) or decrease (quadrant I and III) the single particle normalized amplitude (i.e. the invariant of the motion), depending on the particle phase. In addition, to each transverse kick a corresponding betatronic phase shift is associated. This is illustrated in the scheme of the normalized phase space.

2 Electron lens features

2.1 Inner and outer radius

Given an hollow cathode, the emitted electron beam distribution is different from zero in a region delimited by an inner and an outer radius. While the value of the outer radius can be tuned using different magnetic field values of the central solenoid,

2.2 Radial profile

The current I_r can be written as $I_r = I_T\ f(r)$ where I_T is the total current of the electron beam and the shape function $f(r)$ is defined as:

$$f(r) = \begin{cases} 0 & r < R_1 \\ \frac{r^2 - R_1^2}{R_2^2 - R_1^2} & R_1 < r < R_2 \\ 1 & r > R2 \end{cases} \tag{5}$$

2.3 Current Jitter

2.4 Diffusion

Measurement of a diffusion coeffient have been recently performed at CERN for the LHC machine. It was proposed to include a first approxiamtion of the diffusion process Sixtrack, by providing and emittance change on turn-by-turn basis, assuming a Brownian motion for the particles.

In order to accomodate a diffusive-like process in Sixtrack, an emittance change is applied to each particle at the end of each turn, at the last element of the sequence.

2.4.1 From diffusion coefficient to rms kick: an LHC example

In this case, the diffusion coefficient can be espressed as a function of the average change in Hamiltonian action ΔJ in a time interval Δt, i.e.

$$D =< \Delta J >^2 /(2\Delta t) \tag{6}$$

where the Hamiltonian action associated to a specific collimator half aperture z_c is defined as:

$$J = z_c^2/(4\beta) \tag{7}$$

or alternatevely, in terms of standard emittance definition:

$$J = \varepsilon/4 \tag{8}$$

In case of uncoupled motion, the mentioned quantities can be defined independently for the horizontal and the vertical plane. Starting from the diffusion coefficient we can therefore calculate the average change in emittance per turn:

$$4\,\Delta\varepsilon = \sqrt{2\,D\,\Delta t}$$

$$\Delta\varepsilon = \sqrt{\frac{D\,\Delta t}{8}} \tag{9}$$

From [ref], we have that a typical value of LHC diffusion coefficient is of the order of $1 \cdot 10^{-10} \mu m^2/s$, and the LHC revolution period is about $90\mu s$; inserting these values in the previous equation we obtain the expected average emittance variation per turn:

$$\Delta\varepsilon = \sqrt{\frac{1 \cdot 10^{-10} \mu m^2/s \cdot 90 \cdot 10^{-6} s}{8}} = 3.3 \cdot 10^{-8} \mu m \tag{10}$$

meaning that, for a typical LHC emittance such $\varepsilon_x = 2\mu m$, the relative emittance variation per turn is:

$$\Delta\varepsilon/\varepsilon = 19 \cdot 10^{-9} \tag{11}$$

and considering a rms beam size of 500 um, the rms position variation per turn would be of $19 \cdot 10^{-9} \cdot 500 \mu m \approx 1 \cdot 10^{-5} \mu m$

3 Electron Lens operation modes

3.1 DC mode

Since the electrostatic force is always directed inward, and the electron beam versus is usually chosen such as the Lorentz force adds up to the electric one, the kick given by the electron lens is always directed inwards (focusing), both for the vertical and for the horizontal plane. If we consider what would be the effect of a single kick of the electron lens on the unperturbed harmonic motion of a particle in a linear machine, it is clear that a single inward kick can both increase (quadrant II and IV) or decrease (quadrant I and III) the single particle normalized amplitude (i.e. the invariant of the motion), depending on the particle phase. In addition, to each transverse kick a corresponding betatronic phase shift is associated. This is illustrated in the scheme of the normalized phase space in Figure **??**.

This consideration, which is valid for a single kick uncorrelated with the particle motion, has to be taken very carefully if the kick given by the electron lens is acting constantly on the particle motion (DC mode): in this case the electron lens field becomes a part of the periodic lattice and the single particle invariant for stable particles must be re-defined. The electron lens effect is to introduce a deformation of the phase space together with a tune shift of the particle. The DC mode of the electron lens to introduce a deformation of the particle orbit in the phase space and a tune shift. This has been verified by simulations.

The deformation of the phase space induced by the non linearity of the electron lens has a very mild effect. A maximum amplitude oscillation of the order of few hundredths of sigma is introduced by the electron lens.

The tune shift of the particle is of the order of few 10^{-4} and depends on particles initial conditions, as shown in Figure **??**: as expected the tune shift is always positive (because the electron lens is always focusing) and the tune shift is larger for particles with a larger betatronic amplitude. Since the electron lens kick amplitude depends linearly on the particle position, it follows that the dependency on the tune shift by the initial amplitude is also linear with the particle initial amplitude. The tune shift reaches a maximum value of about 5 10^{-4}. Given the LHC working point, such a tune shift is not sufficient for driving the particle into a high order machine resonance. For each particle the tune jitter for different time windows, which has been studied with a precision of about 1 10^{-5} is negligible, which seems to be a clear indication that the motion is still harmonic. Further studies should be performed with dedicated tools which allow to see higher order resonances [Lifetrack].

3.2 random mode

When the electron lens is used in random mode, the beam current is randomly switched on or off on a turn-by-turn basis.

3.3 AC mode

3.4 resonant

4 Electron lens database

In this paragraph the new entry defined for an electron lens type collimator is described and the meaning

The absolute values of R_1, R_2 can be arbitrary modified by acting on the solenoidal magnetic field that confines the electron beam, but the ratio $g = R_1/R_2$ is fixed by the geometry of the cathode. The electron lens element is represented in the optics input file as a thin element, essentially a marker which is recognized to be an electron lens only by the element name. The description of the different entries is here detailed:

- *name1, name2*: name of the element in the optics sequence. Name1 must be in uppercase and name2 must be written in lowercase;
- *half aperture*: electron lens half inner radius, expressed in normalized units [sigma] in the plane specified as "collimation plane" (see below);
- *lenght*: electron lens active length [m];
- *collimation plane*: designed collimation active plane, expressed as the angle between the horizontal plane and the collimation plane itself [rad];

Table 1: New format for electron lens entry in the collimation database input file.

line						
	TYPE *element 1 description*	TYPE *element 2 description*	... ...			
1	STRING *name1*					
2	STRING *name2*					
3	DOUBLE *half aperture*					
4	DOUBLE *length*					
5	DOUBLE *collimation plane*					
6	DOUBLE *Xcenter*	DOUBLE *Ycenter*				
7	DOUBLE *current*	DOUBLE *voltage*				
8	DOUBLE *R2/R1*					
9	INT *operation mode*	DOUBLE *operation tune*	DOUBLE *Delta tune*	DOUBLE *tune step*	INT *turns per step*	INT *resonant turns*
10	LOGIC *jitter*	LOGIC *radial profile*				
11	DOUBLE *betaX*					
12	DOUBLE *betaY*					

- *x center, y center*: horizontal and vertical coordinates of the electron lens center [m];
- *current*: electron beam total current [A];
- *voltage*: accelerating voltage [eV];
- *R2 to R1 ratio*: ratio between the inner and the outer electron beam

radius;

- *operation mode*: variable defining the electron lens operation mode: 0=DC, 1=random, 2=AC, 3=resonant;

- *operation tune*: central electron lens AC frequency (only for AC mode);

- *Delta tune*: if this variable is different from zero, then the AC frequency will vary between the central operation tune minus the Delta tune and the central operation tune plus Delta tune, in steps defined by the variable *tune step* (only for AC mode);

- *tune step* : tune minimum step which is used to sample the the total electron frequency range (only for AC mode);

- *turns per step*: number of turns which are between two tune steps (only for AC mode);

- *resonant turns*: the electron lens is switched on only if the turs is a multiple of the variable *resonant turns* (only for resonant mode);

- *jitter*: activate or de-activate a 2% jitter in the total electron lens current [TRUE-FALSE];

- *radial profile*: activate or de-activate the measured current radial profile [TRUE-FALSE];

- *beta x - y*: horizontal and vertical beta functions at the electron lenslocations. These values are considered only if the appropriate variable (do_nominal) is selected in the fort.3 input file. [m]